Sigmoid functions

A **sigmoid function**, also called a **logistic function**, is an "S"-shaped continuous function with domain over all $\mathbb{R}$. However, the range is only over $(0, 1)$. It's graph is plotted in Figure 1.

Definition 1 *The* ***sigmoid function****, also called* ***logistic function****, is a function*

$$\sigma : \mathbb{R} \to (0, 1)$$

defined as

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

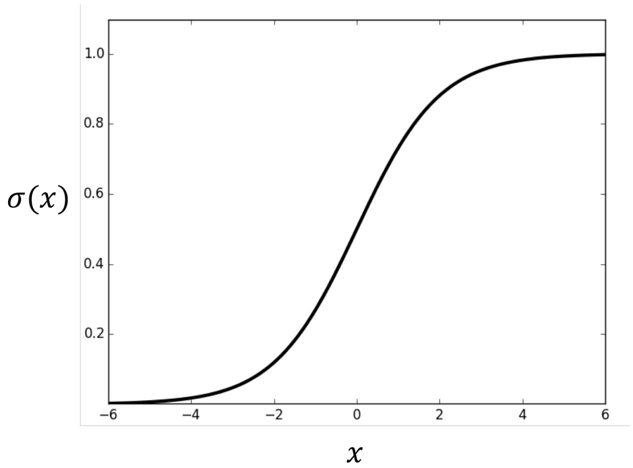

Figure 1: The sigmoid function.

Note that at $x = 0$, the sigmoid is $\frac{1}{1+e^0} = \frac{1}{1+1} = 0.5$. Furthermore,

$$\lim_{x \to \infty} \frac{1}{1 + e^{-x}} = 1$$

and

$$\lim_{x \to -\infty} \frac{1}{1 + e^{-x}} = 0$$

It's first order derivative is

$$\frac{d\sigma(x)}{dx} = \sigma(x)[1 - \sigma(x)]$$

as proven in Theorem 1

Theorem 1

$$\frac{d\sigma(x)}{dx} = \sigma(x)[1 - \sigma(x)]$$

Proof:

$$\begin{aligned}
\frac{d\sigma(x)}{dx} &= -(1 + e^{-x})^{-2}(-e^{-y}) && \text{by the chain rule} \\
&= \left(\frac{1}{1 + e^{-x}}\right)\left(\frac{e^{-y}}{1 + e^{-x}}\right) \\
&= \sigma(x)\left(\frac{e^{-x}}{1 + e^{-x}}\right) \\
&= \sigma(x)\left(\frac{1 + e^{-x}}{1 + e^{-x}} - \frac{1}{1 + e^{-x}}\right) \\
&= \sigma(x)[1 - \sigma(x)]
\end{aligned}$$

□